

Superconductivity Induced by the Proximity Effect of Singlet Resonating Valence Bond Order in High- T_c Superconductors

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Effects of inhomogeneous doping on the high- T_c cuprate superconductors are studied within the framework of the t - J model. Especially, the boundary between two non-superconducting regions with doping rates much higher and lower than the optimal one is examined. It is found that, although there is no superconductivity in the bulk, a superconducting region appears at the boundary because singlet resonating valence bond order and holon condensation occur simultaneously in this region. The critical temperature of the induced superconductivity can be higher than that of the optimally doped sample and the critical current density can be the same order as that in the superconducting state below the critical temperature. We also point out an experimental possibility to observe this phenomenon.

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It has been recognized that there are two important ingredients for the superconductivity in high- T_c cuprates, which are the spin correlation, leading to the spin gap, and the doped charge carriers with a sufficiently high density. In the slave-boson mean-field theory of the t - J model, the former is realized as the singlet resonating valence bond (s-RVB) order of spinons and the latter the bose condensation of holons [1, 2, 3, 4, 5, 6, 7]. In this framework, it is assumed that the spinons and the holons are well-defined low lying excitations, and the spin-charge separation is a good picture. Although there still remains some controversy on this point, it has been clarified that some physical properties of the high- T_c cuprates can be well described by the t - J model, for example, the transport properties [8] and the magnetic properties [9, 10]. In this Letter, we take the t - J model to be our starting point.

We consider the situation where the doping rate is spatially varying. We especially concentrate on the case where two non-superconducting regions, one is too over-doped and the other is too under-doped, are in contact with each other within the same CuO_2 plane. We expect that the s-RVB order and the holon condensate can exist simultaneously at the boundary, thus giving rise to superconductivity. Since the s-RVB order and the holon condensation can occur at temperatures higher than the critical temperature of the optimally doped sample, T_{opt} , this “boundary superconductivity” may also occur above T_{opt} .

Our idea is similar with the “spin-gap proximity effect” previously introduced by Emery, Kivelson and Zacher [11] based on a somewhat different model for high T_c superconductors, discussing the stripe order and its significance for the superconductivity. Recently, Kivelson has proposed a possibility to raise critical temper-

ature using spin-gap proximity effect [12]. He proposes making a stack of spin-gap material and hole-rich material in a layered structure. In contrast to this, we consider the situation where the spin-gap and hole-rich regions exist within a CuO_2 plane. This kind of situation has not been studied within the framework of the t - J model and we consider this is of interest from both theoretical and experimental points of view.

First we introduce the t - J model which describes the correlated electrons in each CuO_2 plane. The Hamiltonian is given by

$$\mathcal{H} = -t \sum_{\langle i,j \rangle \sigma} (f_{i\sigma}^\dagger f_{j\sigma} b_j^\dagger b_i + \text{h.c.}) + J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - \sum_i \lambda_i \left(\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i - 1 \right), \quad (1)$$

where i and j denote lattice points in the CuO_2 plane, $\langle i, j \rangle$ means the nearest neighbors, $f_{i\sigma}$ ($\sigma = \uparrow, \downarrow$) and b_i are the annihilation operators of the spinon and the holon, respectively, \vec{S}_i denotes $\frac{1}{2} \sum_{\alpha\beta} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$ with $\vec{\sigma}_{\alpha\beta}$ being Pauli matrices, and λ_i is the Lagrange multiplier for the constraint, $\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1$. In this Letter the lattice spacing is taken to be unity. In addition to Eq. (1), we later introduce the coupling to the electromagnetic field.

The Eq. (1) is treated by the so-called slave-boson mean-field theory. The local constraint represented by λ_i is replaced by the global one, which is described by the constant chemical potential for spinons μ_F and that for holons μ_B . We define the statistical averages, $\xi \equiv \langle \sum_\sigma f_{i\sigma}^\dagger f_{j\sigma} \rangle$ and $\eta \equiv \langle b_i^\dagger b_j \rangle$, which are assumed to be independent of i and j [13]. The order parameter for the s-RVB order is denoted as $\Delta_{ij} = \frac{3J}{8} \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$, which is equal to $\Delta_x(\vec{r}_i)$ if $j = i + \hat{x}$ and $\Delta_y(\vec{r}_i)$ if $j = i + \hat{y}$ where \hat{x} and \hat{y} indicate the shift by one lattice spacing in positive x - and y -direction, respectively. Although ξ , η and Δ_{ij} should be determined self-consistently in principle, we skip this process for simplicity by just assuming ξ

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and η to be constant. Including also the scalar and vector potential, φ and \vec{A} , and the fictitious gauge fields, a_0 and \vec{a} , which represent the fluctuation around the mean-field solution [8], we obtain the following Lagrangian,

$$\begin{aligned} \mathcal{L} = & \sum_{i\sigma} f_{i\sigma}^\dagger \{ \hbar \partial_\tau - \mu_F + i a_0(\vec{r}_i) \} f_{i\sigma} \\ & + \sum_i b_i^\dagger \{ \hbar \partial_\tau - \mu_B + i(e\varphi(\vec{r}_i) + a_0(\vec{r}_i)) \} b_i \\ & + \sum_i i\varphi(\vec{r}_i) \rho(\vec{r}_i) \\ & - \left(t\eta + \frac{3}{8} J\xi \right) \sum_{\langle ij \rangle \sigma} \left\{ e^{i\theta_{ji}^F} f_{i\sigma}^\dagger f_{j\sigma} + \text{h.c.} \right\} \\ & - t\xi \sum_{\langle ij \rangle} \left\{ e^{i\theta_{ji}^B} b_i^\dagger b_j + \text{h.c.} \right\} \\ & - \sum_i \left[\Delta_x^*(\vec{r}_i) \chi_{ii+\hat{x}} + \Delta_y^*(\vec{r}_i) \chi_{ii+\hat{y}} + \text{h.c.} \right. \\ & \quad \left. - \frac{8}{3J} (|\Delta_x(\vec{r}_i)|^2 + |\Delta_y(\vec{r}_i)|^2) \right], \quad (2) \end{aligned}$$

where $\chi_{ij} = f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow}$ and

$$\theta_{ji}^B = \frac{1}{\hbar} \int_{\vec{r}_j}^{\vec{r}_i} \vec{a} \cdot d\vec{l}, \quad \theta_{ji}^F = \frac{1}{\hbar} \int_{\vec{r}_j}^{\vec{r}_i} \left(\vec{a} + \frac{e}{c} \vec{A} \right) \cdot d\vec{l}. \quad (3)$$

The chemical potentials μ_F and μ_B are determined from

$$\langle f_{i\sigma}^\dagger f_{i\sigma} \rangle = 1 - \bar{\delta}, \quad \langle b_i^\dagger b_i \rangle = \bar{\delta}, \quad (4)$$

where $\bar{\delta}$ is the average holon density in the whole system. The background charge density, originating from doping, is denoted by $\rho(\vec{r}_i)$, whose spatial average equals $-e\bar{\delta}$. In this Letter a_0 and \vec{a} are treated perturbatively on the same line with Refs. [8] and [14].

First we study how the distribution of the holons, namely $\delta(\vec{r}_i)$, is determined under an inhomogeneous doping, which is described by $\rho(\vec{r}_i)$. For this purpose, we neglect the terms including Δ_{ij} and \vec{a} and \vec{A} . Then the spinon and the holon degrees of freedom can be integrated out, giving rise to terms, $\pi_F a_0^2 + \pi_B (a_0 + e\varphi)^2$, where π_F and π_B , respectively, are the polarization functions of spinons and holons. Now the magnitudes of π_F and π_B determine which of the spinon and the holon couples to φ . In this Letter we assume $\pi_F \gg \pi_B$, which may be valid in lower doping region. Then a_0 is almost fixed to zero and φ couples to the holons. Now δ and φ are determined from ρ in a self-consistent way.

We especially consider the following situation. The CuO_2 planes, described by the above Lagrangian, are laid parallel to x - y plane and stacked in z -direction. The region $x < 0$ ($x > 0$) is filled with a s-RVB ordered (hole-rich) material with a low (high) enough doping rate. Both of the regions are non-superconducting. The system is uniform in y - and z -direction. We assume that the doping rate, which is described by $\rho(\vec{r}_i)$, changes at $x = 0$

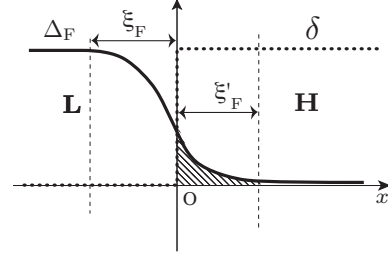


FIG. 1: Spatial variation of the holon density δ (dotted line) and the s-RVB order parameter Δ_d (bold line). **H** and **L** stand for high and low doping region, respectively. In the shaded region the superconductivity is expected to appear.

abruptly. In order to preserve the charge neutrality in the bulk, namely at $x \rightarrow \pm\infty$, the statistical average $\langle \varphi(\vec{r}) \rangle$ must take constant imaginary values, which differ between $x \rightarrow \infty$ and $x \rightarrow -\infty$. (Note that the chemical potential μ_B is a constant in the whole system.) Near the boundary $\langle \varphi(\vec{r}) \rangle$ changes smoothly connecting these two bulk limits. As in the case of semiconductor junctions [15], the smooth connection is enabled by the polarization charge appearing at the boundary. The depth of the charged region is given by the screening length of the holons. This length is extremely long (short) for $x < 0$ ($x > 0$) and there still remains a sharp drop of the holon density at the boundary. Therefore, in this Letter, we approximate δ by a step function (see Fig. 1).

Next we study the s-RVB order by introducing the Ginzburg-Landau (GL) free energy of the order parameter $\Delta_d = \frac{1}{2}(\Delta_x - \Delta_y)$. Here we assume that the s-RVB order has d -wave symmetry. The GL free energy is obtained by the perturbative expansion of the free energy with respect to Δ_d [16]. We introduce the half band width $D = 4(t\eta + 3J\xi/8)$ and the strength of attractive interaction $V = 3J/8$. The dispersion of spinons, $(D/2)(\cos k_x + \cos k_y)$, is approximated by a parabolic one, $D(-1 + |\vec{k}|^2/4)$, and then the density of states at Fermi energy becomes $N(0) = 1/(\pi D)$. At a temperature $T \ll D$, the solution for $\sum_\sigma \langle f_{i\sigma}^\dagger f_{i\sigma} \rangle = 1 - \delta$ is approximately given by $\mu_F = -\delta/N(0)$ and the critical temperature T_d becomes $T_d \simeq T^* \exp[-1/\{2VN(0)(1 - \delta)^2\}]$ with $T^* = 2De^\gamma/\pi$ where γ is the Euler's constant. This expression is valid in the "weak coupling regime", $VN(0) \ll 1$. The GL free energy density of Δ_d is given by

$$F_{\text{s-RVB}} = F_0 + \alpha_d |\Delta_d|^2 + \beta |\Delta_d|^4 + \gamma_d |\vec{\Pi} \Delta_d|^2, \quad (5)$$

where $\vec{\Pi} = -i\nabla + (2/\hbar)\vec{a} + (2\pi/\phi_0)\vec{A}$ with $\phi_0 = hc/(2e)$. The GL coefficients are given by

$$\alpha_d = 2N(0)(1 - \delta)^2 \ln \frac{T}{T_d}, \quad (6)$$

$$\beta = \frac{21\zeta(3)N(0)}{2\pi^2 T^2} (1 - \delta)^4 \equiv \frac{c_1}{T^2 D}, \quad (7)$$

$$\gamma_d = \frac{7\zeta(3)N(0)D^2}{16\pi^2 T^2} \equiv \frac{c_2 D}{T^2}, \quad (8)$$

where $\zeta(x)$ is the zeta function and c_1 and c_2 are constants. The doping dependence of the GL coefficients comes from T_d , α_d and β . Since the largest dependence appears from T_d , we neglect the $(1 - \delta)^n$ -factors in α_d and β in the following. Then the doping rate affects T_d only.

Here we consider the situation where the transition temperatures in the regions $x < 0$ and $x > 0$, denoted by T_d and T'_d , respectively, satisfies the condition, $T_d > T \gg T'_d$. It is also assumed that holons are condensed in the region $x > 0$. Note that the quantity with (without) dash (') is defined in $x > 0$ ($x < 0$). We first determine the spatial variation of Δ_d , disregarding \vec{a} and \vec{A} . Here Δ_d is assumed to be real. The expectation value of Δ_d is $\sqrt{|\alpha_d|/(2\beta)} \equiv \bar{\Delta}_d$ at $x \rightarrow -\infty$ and 0 at $x \rightarrow \infty$. In $x < 0$ we introduce a new variable $\delta\Delta_d = \Delta_d - \bar{\Delta}_d$. The spatial variation of Δ_d is then governed by the following equations,

$$\begin{aligned} 2|\alpha_d|\delta\Delta_d(x) - \gamma_d\partial_x^2\delta\Delta_d(x) &= 0 & (x < 0), \\ \alpha'_d\Delta_d(x) - \gamma_d\partial_x^2\Delta_d(x) &= 0 & (x > 0). \end{aligned} \quad (9)$$

We can easily see that Δ_d behaves as $\bar{\Delta}_d + g_1 \exp(x/\xi_F)$ for $x < 0$ and $g_2 \exp(-x/\xi'_F)$ for $x > 0$, where $\xi_F = \sqrt{\gamma_d/2|\alpha_d|}$, $\xi'_F = \sqrt{\gamma_d/\alpha'_d}$, and g_1 and g_2 are constants. The solution in the whole region is obtained by connecting these two solutions at $x = 0$ with boundary condition,

$$\partial_x\Delta_d(-0) = \partial_x\Delta_d(+0), \quad \Delta_d(-0) = \Delta_d(+0), \quad (10)$$

(see Fig. 1). This condition is in contrast with that for the proximity effect in ordinary superconductor-normal metal junction, where the change of the BCS interaction causes a discontinuity of the gap function [17]. (Note that the quantity which is continuous at the boundary is not the gap function but the anomalous amplitude.) The constants g_1 and g_2 are obtained as

$$g_1 = \frac{\xi_F}{\xi_F + \xi'_F} \bar{\Delta}_d, \quad g_2 = \frac{\xi'_F}{\xi_F + \xi'_F} \bar{\Delta}_d. \quad (11)$$

Note that non-zero Δ_d in $x > 0$ is due to the proximity effect of s-RVB order and ξ'_F gives the proximity length. The temperature dependence of the coherence lengths at $T'_d \ll T \lesssim T_d$ are as follows:

$$\xi_F \simeq \sqrt{\frac{c_2}{2}} \frac{D}{T_d} \frac{1}{\sqrt{1 - T/T_d}}, \quad \xi'_F = \frac{\sqrt{c_2}}{\sqrt{\ln T/T'_d}} \frac{D}{T}.$$

It is interesting to see that ξ'_F is, except for the logarithmic factor, analogous to the ordinary proximity length in the clean limit $\propto \hbar v_F/T$ where v_F is the Fermi velocity. If T is close to T_d , ξ_F becomes much larger than ξ'_F and the amplitude of Δ_d in the proximity region, *i.e.*, g_2 , becomes much smaller than $\bar{\Delta}_d$. However if T is not too close to T_d , ξ_F and ξ'_F can be the same order and g_2 becomes comparable to $\bar{\Delta}_d$. Noting that the holons are already bose condensed in the proximity region, this result means the induced superconductivity in the boundary area.

The strength of the induced superconductivity becomes more apparent by estimating the supercurrent which can flow parallel to the boundary. In this region the gauge field \vec{a} is massive because of the holon condensation, and \vec{a} in Eq. (5) can be set to zero. Then the response to the actual vector potential $\vec{A}(\vec{r})$ is dominated by the spinons. Here we take the vector potential to be constant and parallel to y -direction. The total current in y -direction is given by

$$\begin{aligned} J &= 2c\gamma_d A_y \int_0^\infty \Delta_d^2(x) dx \\ &= c\gamma_d A_y \frac{\{\tilde{\xi}'_F(A_y)\}^3}{\{\tilde{\xi}'_F(A_y) + \xi_d\}^2} \bar{\Delta}_d^2, \end{aligned} \quad (12)$$

where the A_y -dependent proximity length $\tilde{\xi}'_F(A_y) = \xi'_F/\sqrt{1 + p'A_y^2}$ decreases with increasing A_y , with p' being $(2\pi\xi'_F/\phi_0)^2$. The maximum of J as a function of A_y is given approximately by

$$J_{\max} = 2c \left(\frac{2\pi}{\phi_0} \right)^2 \gamma_d \frac{1}{3\sqrt{3}p'} \frac{\xi_F'^3}{\xi_F^2} \bar{\Delta}_d^2. \quad (13)$$

For clarity, here we introduce the critical current density j_c in the bulk superconducting phase. This quantity (in overdoped region) is calculated from Eq. (5) by assuming that holons are condensed and \vec{a} is massive in the whole region. We obtain

$$j_c = 2c \left(\frac{2\pi}{\phi_0} \right)^2 \gamma_d \frac{1}{3\sqrt{3}p} \bar{\Delta}_d^2, \quad (14)$$

where $p = 2(2\pi\xi_F/\phi_0)^2$. It is clear that $J_{\max} = \sqrt{2}j_c\xi_F'^2/\xi_F$. If $\xi'_F \simeq \xi_F$, the critical current density in the proximity region, defined by J_{\max}/ξ'_F , can be the same order as j_c .

Finally we point out some experimental set-ups to observe the effect predicted in this Letter. The simplest way may be to utilize the field effect transistor (FET) recently developed by Shön *et al.* [18, 19]. We propose two types of experiments, which we call “half-gate” experiment and “side-gate” experiment. The experimental set-ups are depicted in Fig. 2. In the former case (Fig. 2(a)) the gate electrode is attached parallel to the CuO₂ plane, but it covers only half of it. As a result, a boundary between the s-RVB ordered region (uncovered by the gate) and the hole-rich region (covered by the gate) is formed under the edge of the gate. (Here we are assuming that the gate induces doping to the undoped compound.) However, in order to observe the effect studied in this Letter, the insulating layer separating the gate and the sample must be smaller than the s-RVB coherence length (approximately the superconducting coherence length below $T_c \sim$ several nm.), which may be hardly realized.

The other configuration is the “side-gate” as depicted in Fig. 2(b). In this case the gate is applied to the CuO₂ plane from the side and by applying a voltage a hole-rich layer is formed under the gate. This situation is a

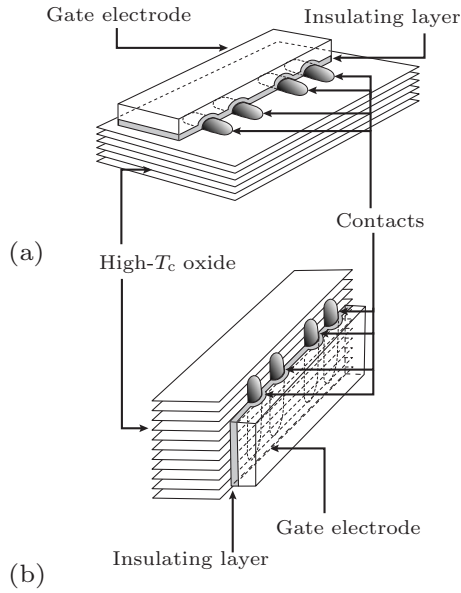


FIG. 2: Experimental set-ups using field effect transistor: (a) half-gate configuration and (b) side-gate configuration. The CuO_2 planes are shown in an exaggerated scale as compared to the electrodes.

little different from one considered in this Letter where the hole-rich region is infinitely wide. However the same mechanism studied by us may also work. In this case, the thickness of the superconducting region is limited not by the s-RVB proximity length but by the thickness of the hole-rich region. This configuration can be realized more

easily.

There may be other ways. One might introduce the spatial variation of doping chemically, for example, by using the epitaxial growth [20]. If this can be realized, it may fit the situation studied in this Letter best.

In either case, the width of the superconducting region is very narrow and the fluctuation is essential[21]. Therefore it is not easy to observe good superconducting behavior. However we expect that whether the superconducting correlation exists or not is an experimentally detectable fact.

In summary, we have studied the effect of inhomogeneous doping in the high- T_c superconductors. We have shown, based on the t - J model, a possibility of superconductivity at the boundary between the s-RVB ordered and the hole-rich regions, caused by the proximity effect of the s-RVB order. We have also suggested experimental set-ups to observe this effect. Since our proposal is based on the spin-charge separation picture, these experiments may also shed a renewed light on the mechanism of high- T_c superconductivity. On the other hand, from a theoretical point of view, the extension of the present study to more general inhomogeneous cases is also important in understanding recently found inhomogeneous phenomena in cuprates [22, 23, 24].

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